Remembering the Mathematics of the Ideal Villa[1]

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The universe of Platonic and Pythagorean speculation was compounded of the simpler relationships of numbers, and such a cosmos was formed within the triangle made by the square and the cube of the numbers 1, 2, 3.... And if such numbers governed the works of God, it was considered fitting that the works of man should be similarly constructed, that a building should be a representative, in microcosm, of the process exhibited at a larger scale in the workings of the world.

– Colin Rowe[2]

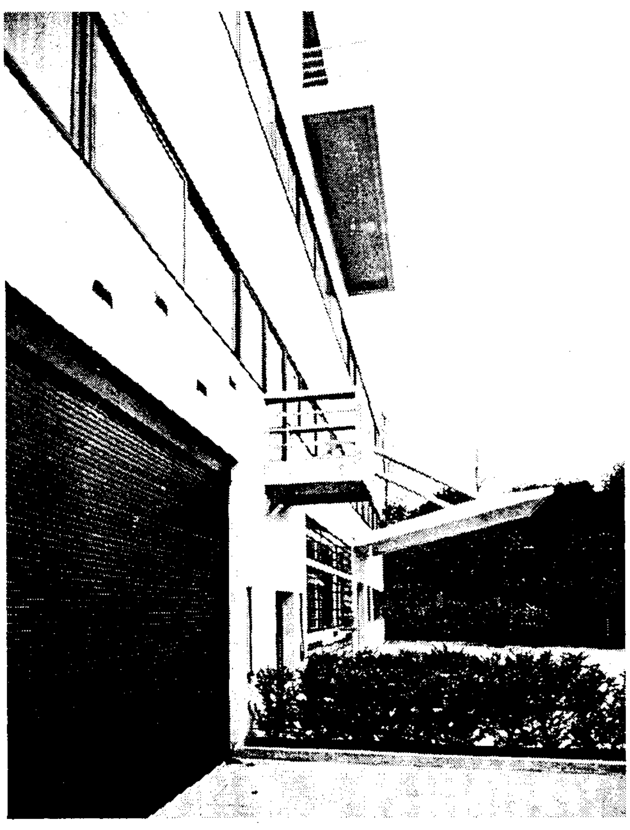


Fig. 1. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. Oblique view of the north facade. From Oeuvre *complète 1910-1929,* 141.

The so-called ABABA rhythm of the structural grid of Le Corbusier's remarkable Villa de Monzie/Stein of 1927 at Garches (Fig. 1) enjoys legendary status in the history of modern architecture. However, the celebrated essay considered to be responsible for explicating the villa's neo-Palladian formula, Colin Rowe's "The Mathematics of the Ideal Villa: Palladio and Le Corbusier compared," first published fifty years ago, actually made no reference to the structural intervals using the designations "A" and "B." The popularization of the formulation ABABA is attributable not to Rowe himself, but rather to commentators on his essay.' In point of fact, following the iconic elevation diagrams Le Corbusier published in his Oeuvre *complète* (Fig. 2). Rowe only applied the designations "A" and "B" in connection with a related but different mathematical property of the villa—namely, the use of the Greek golden section as a geometric proportional device.[5] And, with respect to the rhythm of the structural intervals between the villa's end walls, Rowe was equally strict in employing Le Corbusier's own designations, which are indicated on the same elevation diagrams as 2:1:2:1:2. The numerical sequence, in contrast to the alphabetical one, has the obvious benefit that it not only indicates the alternation of bays but also relates information as to their ratios.

This issue of nomenclature may seem trivial, yet it functions, I believe, as the threshold to a larger debate aimed at heightening perception of the inherent mathematical structure of the Villa de Monzie/Stein's grid, a debate that Rowe's brilliant essay initiated. That is to say, inasmuch as the question of ABABA versus 2:1:2:1:2 is limited to the problem of describing the *transverse* structural intervals, it may also serve to focus attention on a more significant problem: the degree to which the equally important loiigitudinal structural intervals—and thus the essential conditions of the grid as a whole—have been underappreciated.[6] The association of the villa at Garches with the ABABA and/or 2:1:2:1:2 rhythm may be firmly established in the educated architectural mind, but how quickly and surely does one recall the numerical—or alphabetical—sequence of the structural intervals running parallel to the principal axis of the site? Ultimately, in other words, how well does one *remember* the grid?'

The idea of an architecture that is "totally memorable" is characterized by Rowe, quite rightly, as an abstract attribute of no small significance. He introduces the idea in the first sentence of his essay:

> As the ideal type of centralized building Palladio's Villa Capra-Rotunda has, perhaps more than any other house, imposed itself upon the imagination. Mathematical, abstract, four square, without apparent function and totally memorable.[8]

On many levels, Le Corbusier's villa at Garches, an iconic example of modern architecture, is also totally memorable. But the same cannot be said of its celebrated *grid,* at least not the form in which it has been mathematically represented to date. I maintain, however, that if it is considered through a new optic, if the mathematical expression is *transformed* or *defamiliarized,* the grid, rather ironically, is found to be possessed of the essential quality that makes it totally memorable: the quality of the *ideal.*[9] According to this hypothesis, the intervals that define the ordinates of the grid (longitudinal intervals) cannot fail to present themselves as forcibly and enduringly to the mind as the intervals that define the abscissae (transverse intervals) for their mathematical interdependency is shown to be undeniably lucid.

In this paper, I propose a simple alternative numbering system for the grid of the Villa de Monzie/Stein at Garches. Ultimately, the proposal deconstructs the authority of the association of the grid with the simplistic and reductive ABABA/2:1:2:1:2 expressions. In addition to revealing the grid's intrinsic ideality—its nature as an elegant mathematical paradigm, and, accordingly, its mnemonic simplicity—the alternative numbering system also illuminates other fundamental properties of the villa's proportional substructure. These properties focus attention on the power of the longitudinal intervals to regulate the composition of the overall plan, site plan, and underappreciated side elevations.[10]

VILLA DE MONZIE | STEIN'S 1:2:3:4 GRID: A MATHEMATICAL PARADIGM

Rowe's famous analytic diagrams of the Villa de Monzie/ Stein and the Villa Malcontenta, which first appeared in print in 1947, illustrated the schematic interval-structure of the ground-floor plan of Le Corbusier's villa.[11] In the 1976 edition of the article, Rowe illustrated the schematic interval-structure of the *piano nobile* instead of the ground-floor. I have recreated both diagrams here.'? (Fig. 3) One of the significant differences between the transverse intervals and the longitudinal intervals is that the former are contained by the building's rectangular field of enclosure, and hence are finite, although they indeed imply centrifugal extension east and west beyond the end walls via a pattern of infinite alternation (2:1:2:1:2:1:2, and so on). The longitudinal intervals, however, in fact extend beyond the building's primary rectangular field of enclosure, and function to organize the spatial relationships of various secondary and tertiary

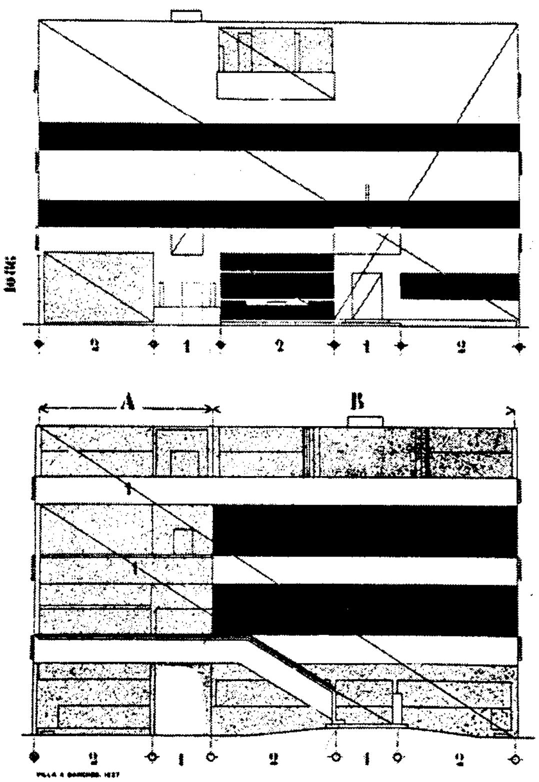


Fig. 2. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. Drawings of the north and south elevations. From **Oeuvre** *complète 1910-1929.* 144.[4]

phenomena (for example, the projected south terrace that Rowe includes in his diagram, to which he assigns the interval designation $1\frac{1}{2}$), which contribute to the all-important organizing function of the longitudinal axis of the site. With this difference in mind, it is still important to consider the grid in the simplest terms as the relationship between the five transverse intervals and the five *major* longitudinal intervals that together describe the building's primary rectangular field of enclosure. Thus, in the Le Corbusier-Rowe numerical system, the transverse sequence is 2:1:2:1:2 and the major longitudinal sequence is $\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:\frac{1}{2}$. What I call the summary sequence—the numbers in ascending order that represent the four different bay sizes or spatial intervals Le Corbusier used in the villa—is $\frac{1}{2}:1:1\frac{1}{2}:2$. These are the four numbers that are now associated in the literature of architecture with the villa's fundamental mathematical structure.

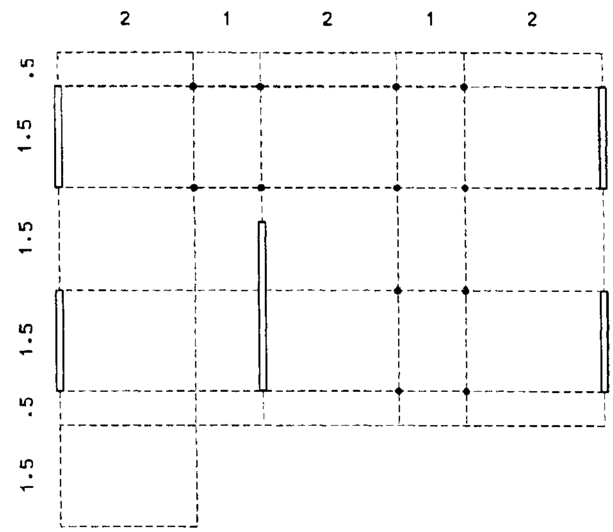
Le Corbusier was clearly sensitive to what mathematicians call "elegance" with reference to an aesthetic property of mathematical assertions. Thus, it is not surprising that he celebrated the 2:1:2:1:2 proportional sequence of the transverse intervals in the *Oeuvre complète.*[13] According to Rudolf Wittkower, the Pythagorean-Platonic tradition regards the 1:2 ratio, which is the ratio of the square to the double square (the point of departure for Le Corbusier's later work on the Modulor), as the basis for all musical consonance: "Perfection and beauty were there ascribed to the ratio itself."[14]

Ultimately, Greek ideals of mathematical perfection and beauty value whole-number relationships. Neither is it surprising, therefore, that Le Corbusier chose to suppress the proportional sequence of the fractional longitudinal intervals, which are not identified in the *Oeuvre complète.* Nor were the side elevations to which they pertain published (though presumably for reasons that have to do with promoting the north and south facades as the primary architectural events of the vertical field).[15] In point of fact, one of the most original aspects of Rowe's essay was that he drew attention to the proportional sequence of the longitudinal intervals, and in so doing, revealed what Le Corbusier had concealed—namely, the *complete* mathematical structure of the grid. But the *inelegance* of the longitudinal sequence that Rowe revealed—$\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:\frac{1}{2}$—presents no small challenge to one's ability to *remember* it, and consequently, to remember the grid as a whole.[16]

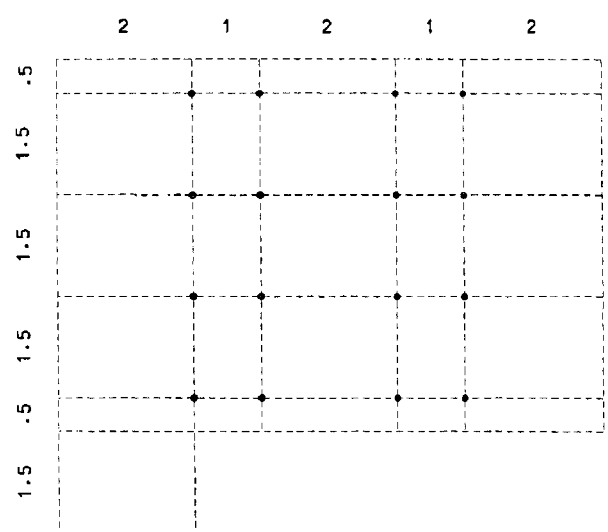
Yet, as Rowe may have intended the reader to infer from his diagram, through the simple mathematical device of *doubling* the numbers, the inelegant fractions are eliminated. According to this alternative numbering system, with respect to the five major intervals that describe the grid in each dimension, the transverse sequence is 4:2:4:2:4 and the longitudinal sequence is 1:3:3:3:1. The summary sequence is 1:2:3:4. This alternative numbering system clarifies the intrinsic elegance of the whole-number relationships of Villa de Monzie/Stein's mathematical structure. The grid is now seen to be ordered by four significant numbers: 1, 2, 3, and 4.[17]

My alternative diagrams depart from Rowe's in other ways as well. They include the suspended entrance canopy to the north and the extra interval of the terrace to the south, from which the outdoor stairway descends to the garden.[18] These additions heighten awareness of the degree to which Le Corbusier regulated the entire plan through the use of this simple 1:2:3:4 relationship. According to this numbering system, then, the primary rectangular volume of the villa oscillates between two readings: (1) at the level of the ground floor, the ratio of the rectangular field is 10:16; (2) at the level of the *piano nobile*, the ratio of the rectangular field is 11:16. Ultimately, Le Corbusier defines the limits of an extended horizontal rectangular field whose ratio is 19:16.

On another level, these alternative diagrams also seek to clarify the structural diagram with respect to the realities of the columnar order.[19] They reveal the difference between the circumstantial disposition of columns at the "profane" ground floor (thirty-one columns) and the more idealized disposition of columns at the "sacred" *piano nobile* (twenty-two columns). Significantly, neither of these floors nor the two upper floors exhibit the ideal condition of twenty-four columns that Le Corbusier's three-bay by five-bay structural matrix would imply—that is, the condition sans cantilevered bays. The five ground-floor columns coplanar with the north facade represent an especially significant example of Le Corbusier's circumstantial deployment of columns. Their presence seems to suggest that the corresponding floor above (the north end



PIANO NOBILE



GROUND FLOOR

Fig. 3. Rowe's diagrams, redrawn by the author, of the schematic interval-structure of the ground floor plan based on Rowe's original illustrations for the 1947 article, and of the *piano nobile* plan based on the illustrations published in Mathematics of the Ideal Villa in 1976. According to the Le Corbusier-Rowe numbering system, the proportional sequences that describe the primary rectangular boundary of the villa are: transverse sequence, 2:1:2:1:2; longitudinal sequence, $\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:\frac{1}{2}$. The summary sequence is comprised of fractional-number relationships, $\frac{1}{2}:1:1\frac{1}{2}:2$. The alphabetical expression, transverse/longitudinal, is ABABA/CDDDC, where B=1.

of the piano *nobile*) is actually cantilevered only at the northwest corner.[20]

Wittkower, on whose scholarship Rowe so heavily depends, wrote that

> all systems of proportion in Western art and architecture ... are ultimately derived from Greek thought. Pythagoras, living in the sixth century B.C., is credited with the discovery that the Greek musical scale depends on the division of a string of the lyre in the ratios 1:2 (octave), 2:3 (fifth), 3:4 (fourth) and 1:4 (double octave). In other words, the ratios of *the first four* integers *1:2:3:4* express all the consonances of the Greek musical *scale*.[21]

As such, the essential 1:2:3:4 (4:2:4:2:4/1:3:3:3:1) structural grid of the villa at Garches indeed constitutes a pure expression of "all the consonances of the Greek musical scale." Far from being in-elegant, Le Corbusier's spatial partitioning reveals itself to be truly in accord with Rowe's description of the Greek ideal—a "universe ... compounded of the simpler relationships of numbers."[22]

In this connection, it is difficult to avoid calling the Parthenon to mind, that "pure creation of the mind,"[23] the symbol of mathematically disciplined plastic perfection that Le Corbusier revered so ardently above all other buildings. Villa de Monzie/Stein represents, ultimately, an elegant mathematical paradigm in the classical Greek sense. Its idealized grid provides the spatial playing field for Le Corbusier's circumstantial game of adding and subtracting columns. But it also functions as the conceptual deep structure of a masterful, prismatic plastic construction. And, if Le Corbusier's employment of the device of the Greek golden section is already central to an understanding of the geometrical construction inherent in the villa at Garches,[24] perhaps we may extend this connection in order to associate the ideal--or unforgettable—mathematics of the villa's structural grid at least as much with the Greek musical/mathematical taxis as with the ABABA rhythm of Palladio's Italian villa.[25]

CANTILEVER/MODULE

The significance of Le Corbusier's assertion that, at Garches, more than any of his other projects, "proportion ruled absolutely there, as absolute mistress,"[26] is underscored by the various implications of this new numbering system. In particular, the degree to which the longitudinal intervals regulate the composition—the overall plan (Fig. 5), the site plan, and the underappreciated side elevations—emerges. This is a point that Le Corbusier's own axonometric studies, which permit rare glimpses of the end elevations, help us to appreciate (Fig. 6), and one that might well be seen to spring from sustained consideration of one idea: the cantilever. On this iconic technological-formal device of modern architecture (and the role it plays) turns more than mathematical differ-

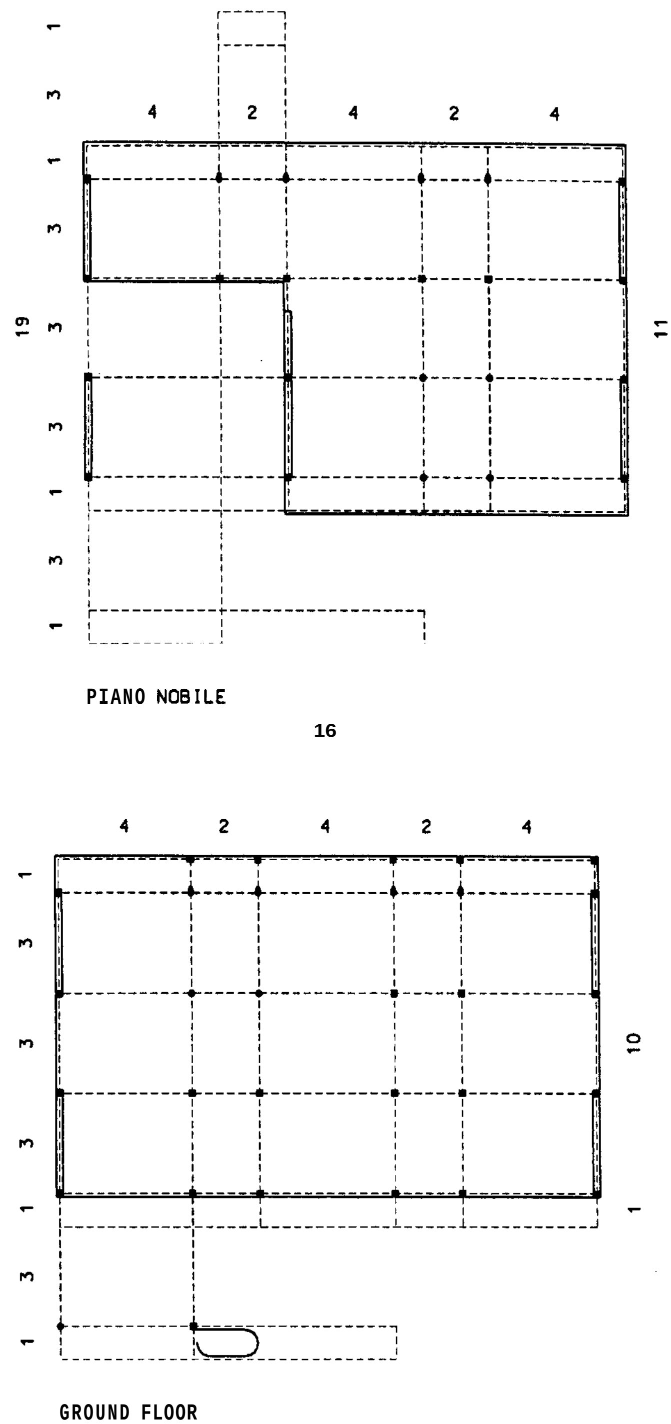


Fig. 4. Diagrams of the author's proposed alternative numbering system. According to this new system, the proportional sequences that describe the primary rectangular boundary of the villa are: transverse sequence, 4:2:4:2:4; longitudinal sequence, 1:3:3:3:1. The summary sequence is comprised of ideal whole-number relationships, 1:2:3:4. (The alphabetical expression, transverse/longitudinal, is ABABA/CDDDC, where C=1; or, DBDBDB/ACCCA, where A=1.) Additional longitudinal intervals that extend south and north into the site are also shown.

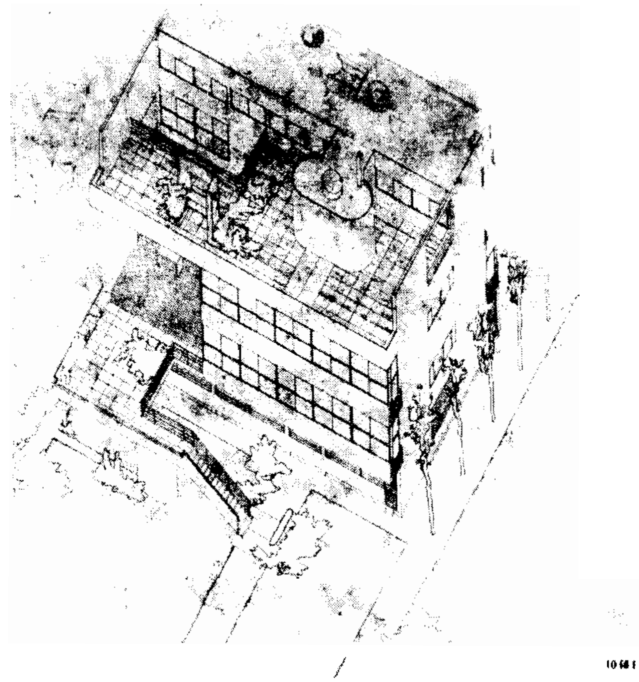


Fig. 6. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. Axonometric view of the southeast comer. FLC 10461, *The Le Corbusier Archive*, vol. 3, 398.

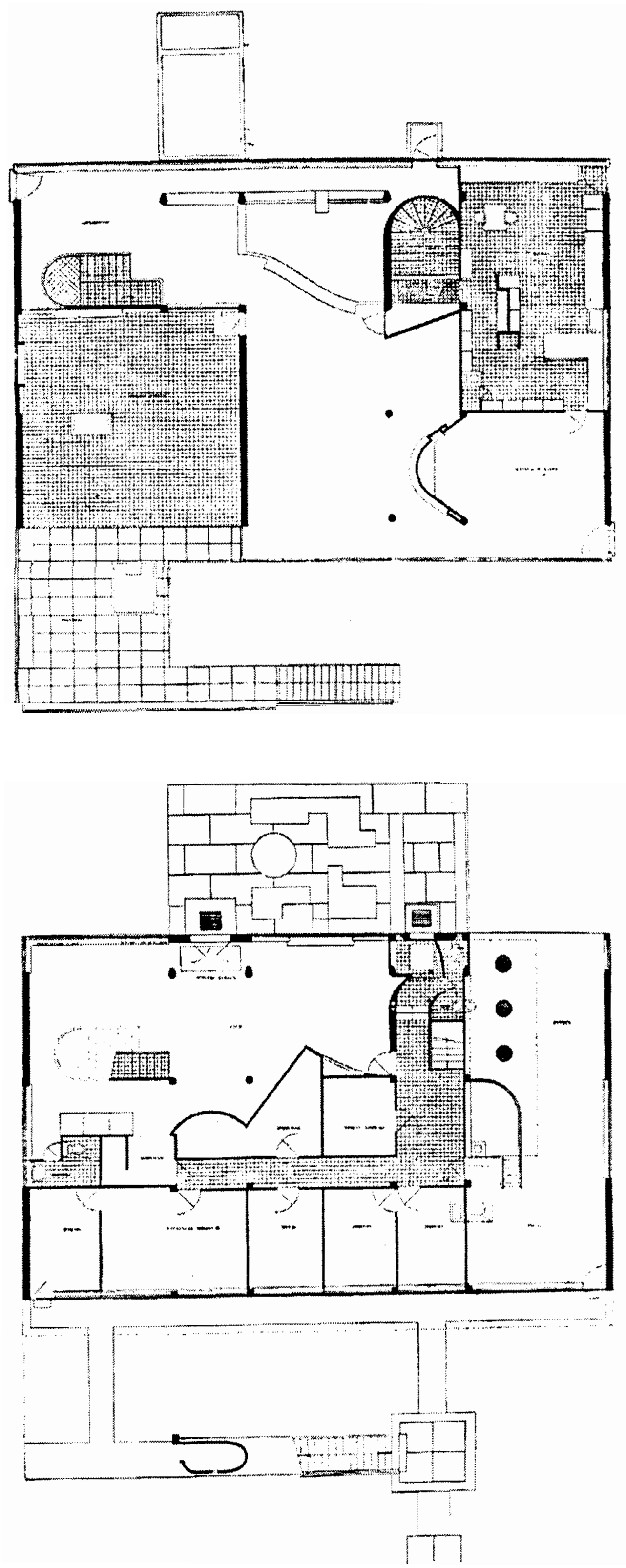


Fig. 5. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. Ground floor plan and *piano nobile* plan. From *Oeuvre complete 1910-1929*, 142. Two free-standing columns in the living room of the *piano nobile*, which do not appear in the *Oeuvrecomplete*, have been added here.

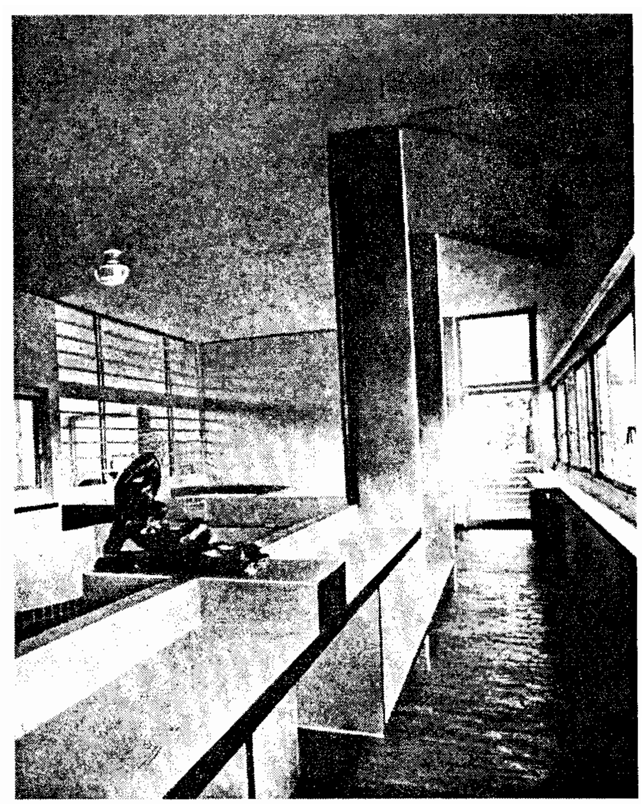


Fig. 7: Le Corbusier, Villa de Monzie/Stein, Garches, 1927. *Oeuvre complète 1910-1929*, 148. View of the interior, showing the spatial significance of the cantilevered interval, 1.25 meters, which is the conceptual starting point ("1") in the author's alternative 1:2:3:4 grid numbering-system.

ences between the two alternative numbering systems. And, in the end, the two numbering systems differ *precisely* on the point that is arguably of greatest significance in one's attempt to appreciate the elegance of Le Corbusier's construction, and which, in the 1:2:3:4 numbering system, is intrinsically associated with the cantilever: that is, the *module*. With respect to the problem of which structural interval of the grid is primary, or generative — that is, which interval ought to be assigned the significant number "1" —the two numbering systems clearly differ on conceptual and pragmatic levels as much as they do on the level of mathematical expression. In the 1:2:3:4 expression of the grid, the cantilevered interval, 1.25 meters, is the starting point "1." (Fig. 7) It is the irreducible basic unit, or module, of which the other intervals are whole multiple moduli.

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NOTES

[1] A longer version of this paper is forthcoming in the *Journal of Architectural Education.*

[2] Colin Rowe, "The Mathematics of the Ideal Villa: Palladio and Le Corbusier compared," *Architectural Review* 101 (March 1947), pp. 101-104. The article has been republished three times. The first was a reprint of the original article, by the same title, in Peter Serenyi, ed., *Le Corbusier in Perspective* (Englewood Cliffs, N.J.: Prentice-Hall, 1975), pp. 46-55. This was essentially identical to the original text; however, the number, selection, and coordination of illustrations were different. The second publication was also a reprint of the original article; it appeared as "The Mathematics of the Ideal Villa," A + U: *Architecture and Urbanism* (October 1975), pp. 29-40 (English text and Japanese translation). Here, the title of the article as well as the compositions and scale of the illustrations differed from both earlier editions. The third re-publication, which comprised a revised text, appeared as the title essay, "The Mathematics of the Ideal Villa," in *The Mathematics of the Ideal Villa and Other Essays* (Cambridge, MA.: The MIT Press, 1976), pp. 1-27. (Since 1978, Rowe's essay has also begun to appear in foreign language translations.) Though the 1976 edition includes significant revisions to the text and illustrations of the original, the essential thesis remains unaltered. I have noted any discrepancies that are pertinent to my thesis. This quotation, which appeared in slightly different form in the original text, is taken from the revised 1976 version.

[3] Numerous scholars have contributed to the popular association of Rowe with the use of alphabetical nomenclature to describe the longitudinal structural intervals of Le Corbusier's villa at Garches. In *Modem Architecture: A Critical History* (New York: Oxford University Press, 1980), for example, Kenneth Frampton, refemng to Le Corbusier's Villa Schwob and the Villa de Monzie/Stein at Garches, writes that "both houses [are] seemingly organized about the *classic Palladian ABABA rhythm remarked on by Colin Rowe,"* p. 157 (my emphasis). In the crucial essay, Rowe does of course associate the villa at Garches with the Palladian model in an indelible way. But it was other scholars who ascribed to this association the so-called Palladian ABABA designation (see n. 7 below). For example, in his article on "The Grid," in *Oppositions* 15/16 (Winter/Spring 1979), p. 102, Barry Maitland refers to Rowe as his point of departure and employs alphabetical notation to show how the "ABABA" transverse-bay spacing at Garches is a special condition of Rudolf Wittkower's diagram of the general Palladian geometrical pattern "ABCBA" where C=A. Interestingly enough, however, Wittkower employs no alphabetical designations; see Wittkower, *Architectural Principles in the Age of Humanism* (London: Alec Tiranti, 1962; reprint ed., New York: W. W. Norton & Company, 1971), p. 73. Nor, for that matter, does Palladio use them in his own *Quattro libri dell'architettura* (Venice, 1570). In fact, we must turn to Cesare Cesariano's 1521 (Como) edition of Vitruvius' *De architectura* for a systematic use of alphabetical notation to describe grid patterns.

[4] Le Corbusier and Pierre Jeanneret, *Oeuvre complète*, 8 vols. (Zurich: Editions d'Architecture, 1929-1970); the following citations are taken from the reprint ed. (Zurich: Editions d' Architecture, 1967).

[5] See the section on the "Villa à Garches 1927," in Le Corbusier, *Oeuvre complète 1910-1929* (vol. 1), pp. 140-149. Rowe describes the device of the Greek golden section, especially in the 1976 version of his essay, as an aspect — together with the associated device of *les trace's régulateurs*—of Le Corbusier's aggressive assertion of the mathematical organization of the elevations: "Le Corbusier ... carefully indicates his relationships by an apparatus of regulating lines and figures and by placing on the drawings of his elevations the ratio of the golden section, A:B=B:(A+B)," *Mathematics of the Ideal Villa*, 9. Rowe supports his view that Le Corbusier's interest in mathematics is most evident in elevation by comparisons with Palladio: "But, if Le Corbusier's facades are for him the primary demonstrations of the virtues of mathematical discipline, with Palladio it would seem that the ultimate proof of his theory lies in his plan," *ibid.*, 9. Elsewhere, Rowe declares "Palladio's choice of plan and Le Corbusier's choice of elevations as being the documents, in each case, most illustrative of elementary mathematical regulation," *ibid.*, p. 12. Part of the purpose of this paper is to show that, in fact, there exists a comparable "elementary mathematical regulation" in Le Corbusier's composition of the plan.

[6] A clarification is in order as to the designations *longitudinal* and *transverse* intervals. Inasmuch as the transverse axis of the building coincides with the longitudinal or north-south axis of the site, and the longitudinal axis of the building coincides with the transverse or east-west axis of the site, a choice must ultimately be made as to which will be privileged — the axes of the building or the axes of the site—in assigning the descriptions "transverse" and "longitudinal" intervals to the complementary dimensions of the grid. I have opted to privilege the axes of the site. This choice offers three interrelated benefits: first, in the most general terms, it underscores the significance of the inextricable spatial/formal relationship of the building and the site; second, on a specific level, it emphasizes the significance of the

underappreciated longitudinal intervals as a major organizing device for the overall site; and third, it serves to illustrate the fact that Garches is as much a contingent, site-dependent fragment of a larger architectural whole as it is an autonomous, site-*in*dependent, Platonic construction. Therefore, I use the term longitudinal intervals to refer to the spatial intervals that run parallel to the long (longitudinal) axis of the site, which runs north-south and is parallel to the short (transverse) axis of the building. The transverse intervals refer to the spatial intervals that run parallel to the short (transverse) axis of the site, which runs east-west and is parallel to the long (longitudinal) axis of the building. Rowe refers to what I call the transverse intervals as the "horizontal coordinates," *Mathematics of the Ideal Villa,* p. 4; indeed, they are horizontal, and thus associated with the x-axis (abscissa) of the building and site plans, when the site plan is oriented with north either up or down, in which case what according to Rowe's system might be called the extended "vertical" coordinates would thus be associated with the y-axis (ordinate) of the building and site plans. I have oriented the plans and diagrams that accompany this article in this way (with north up, as in Rowe's original illustrations to the article). Thus, the contracted horizontal coordinates (or transverse intervals) are read side-to-side along the building and site. And the vertical coordinates (or longitudinal intervals) are read front to back along the building and site. Moreover, though it is somewhat perplexing, the longitudinal intervals, which proceed perpendicular to the two principal facades (north and south), in fact stratify the planar structure of the spatial field parallel to these two facades, for they define the spatial substructure of the deep space of the site's length (its y-axis); and the transverse intervals in fact stratify the planar structure of the spatial field perpendicular to the two principal facade's, for they define the spatial substructure of the shallow width of the site (its x-axis).

7 Analysis of its characterization in the scholarly record reveals the surprising degree to which a mythology has accrued to Rowe's article as well as to the grid of the Villa de Monzie/Stein at Garches itself. In a typical example, in *The Villas of Le Corbusier: 1920-1930* (New Haven: Yale University Press, 1987), Tim Benton writes that *"Rowe emphasized the ABABA grid* of the Villa Stein-de Monzie, comparing it to a similar grid underlying the design of the Villa Malcontenta," p. 165 (my emphasis). First, as I have said, Rowe makes no mention of "ABABA"; second, ABABA is not a "grid," and can only describe a grid if the intervals are the same in both directions, which at Garches they are not; and finally, Benton's assertion actually functions as an example of how other scholars, but not Rowe himself, have "emphasized" the "ABABA" (transverse) intervals of the grid but have lost sight of the longitudinal (what I would call correlatively the "CDDDC") intervals, in other words, the grid itself.

8 Rowe, *Mathematics of the Ideal Villa,* p. 2.

9 Ultimately, Russian Formalism is central to the critical apparatus underpinning my research, namely, the desire to bring a new optic to bear on the familiar object through the technique of *defamiliarization.* Among the seminal texts on the subject is Victor Shklovsky's canonical essay of 1917, "Art as Technique1 Device," which stands as the intellectual cornerstone to my approach. See the English edition, trans. and ed. Lee T. Lemon and Marion J. Reis in *Russian Formalist Criticism: Four Essays* (Lincoln: University of Nebraska Press, 1965).

10 Other commentators have addressed historical, morphological, and constructional aspects of the Garches grid. At least two studies have diligently traced the historical development of Le Corbusier's design of the grid. See Benton, *Villas of Le Corbusier,* 164-189, and Arjan Hebly, "The 5 Points and Form," in Max Risselada, ed., *Raumplan and Plan Libre: Adolf Loos and Le Corbusier, 1919-1930* (New York: Rizzoli Intemational Publications, 1988; reprint ed., 1991), pp. 47-53. Barry Maitland's morphological analysis stands out as one of the unique attempts to examine the grid on a purely theoretical level; see Maitland, "The Grid," pp. 90-117. Attention has been devoted to the related mathematical problem of regulating lines, which, according to Le Corbusier, govern the principal facades. In particular, see Herz-Fischler, "Le Corbusier's 'Regulating Lines' for the Villa at Garches (1927) and Other Early Works," *Journal of the Society of Architectural Historians* 43 (1984), pp. 53-59. An attempt has also been made to relate the regulating lines to the plan in various diagrams. See Yucuru Tominaga and Shigetaka Nagao, "Rediscovery of Modem Housing/Villa à Garches 1927," in *Space Design: A Monthly Journal of Art &Architecture* 133 (September 1975), pp. 66-71. Most relevant to the present study, perhaps, is Herz-Fischler's assertion that "no serious study of Le Corbusier can be based on [his] 'official' versions alone," "Le Corbusier's 'Regulating Lines'," p. 57. For a cogent discussion of the constructional aspects of the Dom-ino reinforced concrete frame system, see Eleanor Gregh, "The Dom-ino Idea," in *Oppositions* 15/16 (Winter/Spring 1979). pp. 60-87.

11 Rowe's diagrams as published in the original text of the essay, "The Mathematics of the Ideal Villa," pp. 102-103, are very difficult to reproduce photographically (one is hard-pressed to decipher the almost illegible interval numbers), but I plan to include them in the forthcoming version of this article.

12 As published in *Mathematics of the Ideal Villa,* p. 5, fig. 1, Rowe's diagram of the villa at Garches is oriented with north to the right. This achieves the effect of placing greater emphasis on the longitudinal intervals, since they are the ones that are read from left to right in this orientation. However, Rowe informed me that this orientation was merely the result of an arbitrary decision made by the publisher.

13 Herz-Fischler asserts that Le Corbusier refers to the 2:1:2:1:2 sequence in his writings as an "automatic system of proportioning," one that he used in many projects, including "Maison Cook, Pessac, and House C-2 at Stuttgart," "Le Corbusier's 'Regulating Lines'," p. 57.

14 Rudolf Wittkower, "Le Corbusier's Modulor," in Carlo Palazzolo and Riccardo Vio, eds., *In the Footsteps of Le Corbusier* (New York: Rizzoli International Publications, 1991), p. 13.

15 The extent to which Le Corbusier himself privileges the transverse, in contradistinction to his silence on the longitudinal, is reflected in this statement from the introduction to the section on Garches in his *Oeuvre complète 1910-1929: "La Maison est entitrement supportée par des poteaux disposés a équi-distance de 5 m et 2 m 5 sans souci du plan intérieur* (The house is supported entirely by columns that are positioned equidistant from one another at intervals of 5 and 2.5 meters throughout the interior)" (my translation), 140. Le Corbusier neglects to clarify that this is true only with respect to the transverse spacing; therefore, he implies that the 5 and 2.5 meter intervals are used to determine the longitudinal placement of the columns as well, which, of course, is not the case.

16 Empirical evidence has helped to corroborate this. In the course of conducting my research, I have observed that both students and faculty colleagues are able to recall accurately either the ABABA or 2:1:2:1:2 aspect of the grid, yet struggle in vain to recall correctly the longitudinal aspect. I have given a number of quizzes to test students' memory/comprehension of the grid after they have studied Rowe's essay. In my experience, only one student has been able to diagram the grid accurately beyond the proverbial ABABA/2:1:2:1:2 rhythm of the transverse structure. A common error, for example, is to employ some combination of A and B for the longitudinal Intervals as well. (Curiously enough, students tend more toward the use of alphabetical than numerical notation.)

17 In discussions with students and colleagues over the years, when the subject of the Garches grid has come up for discussion, no one

has independently raised the idea of *doubling* advanced here. Thus my empirical evidence has corroborated what my study of the published record has revealed, namely, that while it may ultimately be obvious, to my knowledge, there is no evidence that the device of doubling has been previously noted in print or employed in praxis. I made the discovery in 1989 in the course of teaching at the University of Texas (Arlington), where I conducted a graduate studio that involved an addition to the villa at Garches.

[18] Rowe obviously delimits the projection of the terrace in his analytical diagram in order to reinforce the comparison with the portico of the Villa Malcontenta, but he leaves the door open to confusion. The fact that the terrace actually extends an additional ".5" interval in the Le Corbusier-Rowe numbering system (this half interval also defines the zone of the stair that descends to the garden) appears to be an underappreciated point by many who have reprinted Rowe's diagrams. Among the authors whose discussions I have studied, only Kenneth Frampton aligns the diagram and plan of Garches in a manner that accurately and unambiguously reveals the incompletion of the terrace: first in "Frontality vs. Rotation," in *Five Architects: Eisenman, Graves, Gwathmey, Hejduk, Meier* (New York: Wittenborn &Company, 1972; reprint, New York: Oxford University Press, 1975), p. 11; and subsequently in *Modem Architecture: A Critical History,* p. 157. Curiously, however, in another article, Frampton provides an example of the more common phenomenon, wherein the situation is ambiguous and misleading, if not explicitly incorrect; see "Le Corbusier and 'l'Esprit Nouveau'," *Oppositions* 15/16 (Winter/Spring 1979), p. 41.

[19] My diagrams seek to represent the structural condition of the ground floor and *piano nobile* plans, based on Le Corbusier's drawings reproduced in *The Le Corbusier Archive,* ed. *H.* Allen Brooks, 32 vols. (New York: Garland Publishing Company & Paris: Fondation Le Corbusier, 1982-85), hereafter referred to as *LCA,* with drawings from the Fondation Le Corbusier hereafter referred to as FLC with relevent inventory numbers. See, for example, *LCA* vol. 3, *Le Corbusier: Palais de la Société des Nations, Villa Les Terrasses, and Other Buildings and Projects, 1926-1927*, ground floor plans FLC 10576, 452; FLC 10431, 382; and FLC 10451,393. In the same volume, see *piano nobile* plans FLC 10563, 443; and FLC 10452, 393. The aforementioned plates are conclusive with respect to column locations, however they exhibit ambiguities with respect to column shapes, especially in the case of the inside two columns at the service entrance on the ground floor and their expressions at the *piano nobile*.

[20] To my knowledge, only James Michael Ward, in his doctoral dissertation, "Le Corbusier's Villa 'Les Terrasses' and the International Style," New York University, 1984, has observed that the house has, as he writes, "thirty-one vertical supports," which agrees with my decipherment of Le Corbusier's ground floor plan reproduced in *LCA,* vol. 3. But Ward does not comment on the most stunning implication of this discovery, which is that Le Corbusier, by whatever mix of practical necessity and calculated artifice, violates the dictates of his idealized Dom-ino system and presents only the illusion of a cantilevered *piano nobile* at the front facade (the absence of columns at the ribbon window merely signifies that the floor above is cantilevered); at the back, however, by way of physical and conceptual opposition, the facade of the ground floor is effectively subtracted/denied and the reality of the structural act of the cantilever is unequivocally presented; see Ward, "Le Corbusier's Villa 'Les Terrasses'," pp. 178 and 207 n. 45. It should be noted as well that Ward's research, *ibid.,* pp. 19 and 53-56, uncovers the surprising fact that it was Gabrielle de Monzie, co-tenant with Sarah and Michael Stein, who held legal title to the land and assumed principal fiduciary responsibility for the construction of the villa. "Villade Monzie" or "Villa de Monzie/Stein," therefore, as opposed to Villa Stein or even Villa Stein/de Monzie, more properly reflects the historical record. Le Corbusier evidently named the villa Les Terrasses or The Terraces, though he does not refer to it as such in the *Oeuvre complète*. In fact, his drawings bear the designation "Mme G de Monzie," clearly indicating that Le Corbusier knew who was paying the bills.

[21] Wittkower, "Le Corbusier's Modulor," p. 12 (my emphasis); see Rowe, *Mathematics of the Ideal Villa,* 17 n. 6, where he explicitly attributes his observations on the relationship between mathematics, musical harmony, and ideal proportion to his reading of Wittkower.

[22] Rowe, *Mathematics of the Ideal Villa,* 8.

[23] For Le Corbusier's veneration of the Parthenon, see in particular the chapter "Architecture, Pure Creation of the Mind in Le Corbusier, *Towardsa New Architecture,* trans. Frederick Etchells (London: Architectural Press, 1927; reprint, New York: Praeger Publishers, 1960), pp. 185-207.

[24] Two examples of how Garches is typically understood in terms of the golden section, which can be deduced from study of the plates Le Corbusier published in the *Oeuvre complète*, are as follows. (1) As indicated earlier, he includes the basic proportional assertion of the golden section on his drawing of the south facade, A:B=B:(A+B), to indicate that he has used this to organizeits fundamentalleft-right (solid/void) proportional relationship. Thus, according to the transverse intervals along the bottom of his drawing, A=2+1=3, and B=2+1+2=5. Therefore the ratio A:B=3:5. And the ratio B:(A+B)=5:8. The actual proportion of the golden section, which can be derived geometrically and arithmetically, is approximately .618 (a unique property of the Greek golden section is that its reciprocal is the same number added to 1, i.e., 1.618). This is an irrational number that whole number ratios can only approximate, for example, the ratio of thenumbers 3:5=.6, and the ratioofthenumbers 5:8=.625. The ratio 5:8, which more nearly approximates the actual number, is the most common rationalized (that is, approximate) expression of the golden section, and one that Le Corbusier routinely used. (2) The regulating presence of the golden section is also evident in the ratio of the sum of the transverse structural intervals to the sum of the longitudinal intervals of the enclosed part of the ground floor plan (that is, the ratio of the short side to the long side of the plan). In the Le Corbusier-Rowe system, the sum of the longitudinal intervals (from north to south) is 1/2+1 1/2+1 1/2+1 1/2=5. The sum of the transverse intervals is 2+1+2+1+2=8. Thus their ratio is 5:8. This means that the enclosed rectangular field of the ground floor plan conforms to the conventional, idealized ratio of the golden section. And, in point of fact, so does the rectangular vertical field of the north and south facades: the ratio of height to length is also 5:8. (In my doubled numbering system the ratio is correspondingly doubled and expressed as 10:16.) Rowe refers to this important approximation of the golden section—5:8—when describing the similarities between the volumetric proportions of Palladio's Villa Malcontenta and the Villa de Monzie/Stein at Garches: "both are blocks of corresponding volume, each measuring 8 units in length, by 5 1/2 in breadth [this is at the *piano nobile*; the ground floor plan is 5 in breadth], by 5 in height," *Mathematics of the Ideal Villa,* p. 4. This is an example of the way in which Rowe's text has reinforced the association of the golden section with the proportional structure of the villa at Garches.

[25] For an explanation of Aristotle's idea of *taxis* as "the orderly arrangement of parts," which includes the double-idea of the grid and tripartition, see Alexander Tzonis and Liane Lefaivre, *Classical Architecture: The Poeticsof Order* (Cambridge, Mass.: The MIT Press, 1986), p. 9.

[26] From an interview with Le Corbusier by Hugues Desalle, transcribed from an audio recording made six months before the architect's death and published in *Modulus/The University of Virginia School of Architecture Review* (1979), p. 71.